

Exponential Functions

Property of equality for exponential functions: If $b^x = b^y$, then $x = y$

If bases are equal, then the exponents can be set equal. This is one way to solve exponential equations.

Steps for Solving Using Common Bases:

1. Find a common base for both sides of the equation.
2. Rewrite each base as a power of the common base.
3. Simplify the exponent expression if necessary. (Remember: power to a power is multiplied.)
4. Set the exponents equal to each other to make a new equation.
5. Solve the new equation.

Examples:

A. $2^{3n-9} = 64$

$2^{3n-9} = 2^6$

$3n - 9 = 6$

$3n = 15$

$n = 5$

B. $81 = 27^{m-1}$

$3^4 = (3^3)^{m-1}$

$4 = 3m - 3$

$7 = 3m$

$m = \frac{7}{3}$

C. $16^{2n+1} = \frac{1}{32}$

$(2^4)^{2n+1} = 2^{-5}$

$8n + 4 = -5$

$8n = -9$

$n = -\frac{9}{8}$

But... what if there is no common base? Like $6^x = 42$? In this case, logarithms must be used to solve...

Steps for Solving Using Logarithms:

1. Isolate the exponential expression, if necessary.
2. Take the log of both sides. Use ln for problems with e.
3. Move the exponent out in front of the log expression. (Power property of logarithms.)
4. Divide both sides of the equation by the log. (Except when using ln e.)
5. Continue to solve for x, if necessary.
6. Write answers in exact form (with logs), and also as a decimal approximation (round to 3 places).

Examples:

D. $6^x = 42$

$\log 6^x = \log 42$

$x \frac{\log 6}{\log 6} = \frac{\log 42}{\log 6}$

$x = \frac{\log 42}{\log 6}$

$x = \frac{\log 42}{\log 6}$

E. $7^{2x} = 56$

$\log 7^{2x} = \log 56$

$2x \frac{\log 7}{\log 7} = \frac{\log 56}{\log 7}$

$x = \frac{\log 56}{2 \log 7}$

$x = \frac{\log 56}{2 \log 7}$

F. $8^{x+4} - 4 = 100$

$8^{x+4} = 104$

$\log 8^{x+4} = \log 104$

$(x+4) \frac{\log 8}{\log 8} = \frac{\log 104}{\log 8}$

$x+4 = \frac{\log 104}{\log 8}$

$x = \frac{\log 104}{\log 8} - 4$

$x = \frac{\log 104}{\log 8} - 4$

Solving Exponential Equations

Solve using logarithms. Give the exact answer and the answer to 3 decimal places.

1. $2^x = 53$

2. $8^{2x} = 124$

3. $4^{x+5} = 7$

4. $2^{x-5} = 9$

5. $e^x = 10$

6. $4e^{2x} = 5$

7. $7 - e^x = 5$

8. $-14 + 3e^x = 7$

9. $7^x = 20$

10. $9e^{5x} - 4 = 50$

11. $9^{4x} = 25$

12. $6^{x+2} = 17$

13. $7^{x-4} = 8$

14. $e^x = 72$

15. $2e^{12x} = 17$

16. $e^x + 5 = 60$

17. $100e^{5x} = 750$

18. $8^x = 4$

19. $3^{4x} = 3^{x-5}$

20. $5^{x-1} = \frac{1}{25}$

21. $7^{2x} = \frac{2}{49}$