## Properties of Logarithms:

| | Condensed Form | | Expanded Form |
|---|---|---|---|
| Product Property | $\log_b mn$ | $=$ | $\log_b m + \log_b n$ |
| Quotient Property | $\log_b \frac{m}{n}$ | $=$ | $\log_b m - \log_b n$ |
| Power Property | $\log_b m^p$ | $=$ | $p \cdot \log_b m$ |
| Equality Property: $If \ \log_b m = \log_b n \ then \ m = n$ | | | |

**Examples:**

**Use the properties of log to expand each log:**

1. $\log 3xy$

$\log 3 + \log x + \log y$

2. $\log \frac{5x}{y}$ $\log 5 + \log x - \log y$

3. $\log 7a^2b$

$\log 7 + 2\log a + \log b$

4. $\log \frac{9a^2}{cb^3}$

$\log 9 + 2\log a - \log c - 3\log b$

**Use the properties of log to write each as a single log:**

5. $\log_2 x + \log_2 y - \log_2 z$ $= \log \frac{xy}{z}$

6. $3\log_2 x - \log_2 y + 4\log_2 z$ $= \log \frac{x^3 z^4}{y}$

7. $2\log xy + \log 5 - 3(\log z + \log 2)$ $= \log \frac{5x^2y^2}{8z^3}$

**Proof of the Product Property:**

*Let* $x = \log_b m$ *and* $y = \log_b n$

*then* $b^x = m$ *and* $b^y = n$

$b^x b^y = mn$

$b^{x+y} = mn$

$\log_b mn = x + y$

$\log_b mn = \log_b m + \log_b n$

**Proof of the Quotient Property:**

*Let* $x = \log_b m$ *and* $y = \log_b n$

*then* $b^x = m$ *and* $b^y = n$

$\frac{b^x}{b^y} = \frac{m}{n}$

$b^{x-y} = \frac{m}{n}$

$\log_b \frac{m}{n} = x - y$

$\log_b \frac{m}{n} = \log_b m - \log_b n$

Change of base formula: $\log_b m = \frac{\log_n m}{\log_n b}$

*Note:* the base goes on the bottom

*Hint:* the new base "n" is usually 10 or e, because those bases can be evaluated with a calculator

**Evaluate (give the calculator ready answer, then find the value using your calculator):**

8. $\log_4 22$

$\frac{\log 22}{\log 4}$

2.23

9. $\log_{12} 95$

$\frac{\log 95}{\log 12}$

1.83

Proof of the change of base formula:

*Let* $\log_a n = x$ *then* $a^x = n$

$\log_b a^x = \log_b n$

$x \log_b a = \log_b n$

$x = \frac{\log_b n}{\log_b a}$

$\log_a n = \frac{\log_b n}{\log_b a}$