

Exponential Equation - an equation with a variable in the exponent

To solve an exponential equation:

- > Use the calculator to graph each side of the equation as a separate function. Find the intersection of the graphs. (good for estimating solutions)

-or-

- > If you can make the bases the same, do it. (gives an exact answer)

If  $b^m = b^n$  then  $m = n$

-or-

- > If you can NOT make the bases the same, use logarithms. (use log or ln on the calculator)

Take the log of both sides (base 10 or base e)

Examples:

1.  $4^x = 24$

$\log_4 4^x = \log_4 24$   
 $x = \log_4 24$

$\log_7 7^x = \log_7 20$   
 $x = \log_7 20$

2.  $7^x = 20$

$x = \log_7 20$

3.  $13^x = 5^{x+2}$

(3)  $\log 13^x = \log 5^{x+2}$   
 $x \log 13 = x \log 5 + 2 \log 5$   
 $-x \log 5$   
 $x (\log 13 - \log 5) = 2 \log 5$   
 $x = \frac{2 \log 5}{\log 13 - \log 5}$

(4)  $\log 7^{x-2} = \log 5^{3-x}$   
 $x \log 7 - 2 \log 7 = 3 \log 5 - x \log 5$   
 $x \log 7 + x \log 5 = 3 \log 5 + 2 \log 7$   
 $x (\log 7 + \log 5) = 3 \log 5 + 2 \log 7$   
 $x = \frac{3 \log 5 + 2 \log 7}{\log 7 + \log 5}$

4.  $7^{x-2} = 5^{3-x}$

$x (\log 7 + \log 5) = 3 \log 5 + 2 \log 7$   
 $x = \frac{3 \log 5 + 2 \log 7}{\log 7 + \log 5}$

Examples:  
Write in logarithmic form.

1.  $4^3 = 64$

$\log_4 64 = 3$

Write in exponential form.

3.  $\log_{10} 0.01 = -2$

$10^{-2} = .01$

Evaluate

5.  $\log_2 8$

$-2$

7.  $\log_2 32$

$5$

9.  $\log_6 1$

$0$

Solve each equation.

11.  $\log_5 25 = x$

$5^x = 25$

$x = 2$

13.  $\log_{10} x = -1$

$10^{-1} = x$

$x = \frac{1}{10}$

15.  $\log_2 4 = \frac{1}{2}$

$2^{\frac{1}{2}} = 4$

$x = 16$

2.  $125^{\frac{1}{3}} = 5$

$\log_{125} 5 = \frac{1}{3}$

4.  $\log_7 49 = 2$

$7^2 = 49$

6.  $\log_{10} 1000$

$3$

8.  $\log_2 64$

$2$

10.  $\log_3 27$

$\frac{3}{2}$

12.  $\log_2 x = -3$

$2^{-3} = x$

$x = \frac{1}{8}$

14.  $\log_{10} x = \frac{2}{3}$

$10^{\frac{2}{3}} = x$

$x = 4$

16.  $\log_3 243 = x$

$3^x = 243$

$x = 5$